Improved Heat Transfer Correlations for Quantifying Laminar Natural Convection Across Fenestration Glazing Cavities

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ABSTRACT

A number of correlations for predicting convective heat transfer in glazing cavities have been developed over the years, but the most popular and most widely used, especially in North America, have been the correlations from ElSherbiny et al. (1982). This was one of the first correlations to include the Nusselt number dependence on the aspect ratio (cavity height to width). However, this dependence applies only for aspect ratios below 20, which are not representative for typical fenestration systems (windows and doors) that usually have higher aspect ratios. Experimental and analytical studies indicate that the Nusselt number dependence on the aspect ratio extends to much higher aspect ratios, although not as strongly as for aspect ratios less than 20.

In this work, two-dimensional laminar natural convection heat transfer of air-filled, high aspect ratio rectangular cavities, subject to constant temperature boundary conditions on the vertical sides and zero heat flux boundary conditions on the horizontal surfaces, was analyzed using a finite element computational fluid dynamic solution method. The numerical analysis was performed over a range of aspect ratios from A = 5 to 110 for Rayleigh numbers within the laminar flow regime, which covers typical design conditions for fenestration systems and solar energy collectors. Based on the numerical results obtained, a new set of improved heat transfer correlations is developed as functions of both the Rayleigh Number (Ra_L) and aspect ratio (A). The new correlation resolves the dependence of the averaged Nusselt number (Nu_L) on higher aspect ratios. In addition, the new correlations apply not only to lower Rayleigh numbers and higher aspect ratio glazing cavity fenestration products but are also applicable to systems with lower aspect ratio cavities.

INTRODUCTION

Fenestration systems (windows, doors, skylights, and curtain walls) are possibly the most complex and interesting components in commercial and residential building envelope design. Fenestration systems provide exterior views, fresh air when seasonable, and natural light to the interior spaces of the building. However, fenestration systems also represent one of the largest sources of unwanted heat gain in summer and significant heat loss in winter through the building envelope. To provide more accurate and consistent energy performance information of a fenestration system for consumers as well as manufacturers, the National Fenestration Rating Council (NFRC) developed a rating system based on the whole product performance that may be obtained from test laboratory measurements or computer (analytical) calculations. In conjunction with the efforts of NFRC, the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) has proposed standard methods for quantifying the heat transfer and total optical properties of fenestration products (ASHRAE 1996). One of the most important energy performance indices for the insulating value of fenestration products is the overall U-factor, which is, in turn, calculated by separate U-factors at various fenestration product sections (e.g., center-of-glass, edge-of-glass, frame, divider, and divider edge-of-glass), which are combined through an area-weighted method. Edge-of-glass, frame, divider, and divider edge-of-glass U-factors are determined in terms of the heat transfer rate through the frame/edge-of-glass and divider/divider edge-of-glass using numerical modeling. The center-of-glass U-factor is the reciprocal of the center-of-glass thermal resistance, which is calculated by the following equation:

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$$R_{cg} = \frac{1}{h_o} + \sum_{j=1}^{n} \frac{d_j}{k_j} + \sum_{j=1}^{n-1} \frac{1}{h_{gj}} + \frac{1}{h_i} \quad (1)$$

where

d_j = the thickness of jth glass pane,

k_j = the thermal conductivity of the jth glass pane,

h_i = the overall surface heat transfer coefficient for the combined convective and radiant heat transfer on the indoor fenestration surface,

h_o = the overall surface heat transfer coefficient for the combined convective and radiant heat transfer on the outdoor fenestration surface,

h_{gj} = the overall cavity heat transfer coefficient for the combined convective and radiant heat transfer in the gap space within the jth glazing cavity.

Correlations for quantifying the convective heat transfer rate through air-filled cavities has been a continuous research subject due to its significant role in evaluating the overall thermal performance of fenestration products. The evolution of heat transfer correlations derived either experimentally or numerically over the past 40 years was recently reviewed by Zhao et al. (1999). At present, the rate of convective heat transfer in the gas-filled glazing cavity is usually obtained from correlations derived from experimental data. In the United States, there are two computer programs, THERM (LBNL 1996) and WINDOW4.1 (LBNL 1993), that use empirical correlations to evaluate the glazing cavity convective heat transfer coefficient, which is then used in the calculation of the center-of-glass U-factor. A detailed finite element calculation of the two-dimensional heat transfer within the glazing cavity is also provided by the THERM program using an effective cavity thermal conductivity that is based on the glazing cavity convective heat transfer coefficient and a radiant heat transfer coefficient. The current experimentally derived convective heat transfer correlation used in THERM and WINDOW4.1 is based on the work of ElSherbiny et al. (1982) as shown in Equation 2:

$$Nu_L = \begin{bmatrix} 0.0605 Ra_L^{1/3} \\ \{1 + [0.104 Ra_L^{0.293}/(1 + (6310/Ra_L)^{1.36})]^3\}^{1/3} \\ 0.242 \cdot (Ra_L/A)^{0.272} \end{bmatrix}_{max} \quad (2)$$

where

Nu_L = cavity Nusselt number,

Ra_L = cavity Rayleigh number,

A = cavity aspect ratio (cavity height to width).

Several shortcomings exist in ElSherbiny's correlation. The first equation, $0.0605Ra_L^{1/3}$ underpredicts the cavity convective heat transfer rate for low Rayleigh numbers (Ra < 5000). It also inadvertently clips the second equation, and as a result, a discontinuity exists in the correlation. In addition, although the third equation of ElSherbiny's correlation takes the aspect ratio into account, a detailed examination shows that it only affects the Nusselt number (Nu_L) if the aspect ratio (A) is less than 20, and its effect becomes less as A approaches 20. An improved correlation, which is currently adopted by ASHRAE SPC-142P (ASHRAE 1996), was developed by Wright (1996) and has the form of:

$$Nu_L = \begin{bmatrix} \left(Nu_{L1} = \begin{bmatrix} 0.0673838 Ra_L^{1/3} for Ra_L > 5 \times 10^4 \\ 0.028154 Ra_L^{0.4134} for 10^4 < Ra_L \le 5 \times 10^4 \\ 1 + 1.75967 \times 10^{-10} Ra_L^{2.2984755} for Ra_L < 10^4 \end{bmatrix} \right) \\ Nu_{L2} = \left(\frac{Ra_L}{A}\right)^{0.272} \end{bmatrix}_{max} \quad (3)$$

This correlation was based on the experimental data of both ElSherbiny et al. (1982) and Shewen (1986). It differs from ElSherbinyElSherbiny's correlation in that the clipping problem was avoided and the applicable aspect ratio range was increased from $A < 20$ in Equation 2 to $A < 25$ in Equation 3. For A > 25, both ElSherbiny's and Wright's correlations do not include the dependence of Nusselt number on aspect ratio. It has long been recognized that the aspect ratio has a significant effect on the Nusselt number for higher aspect ratios. It is believed by the authors that the experimental data in ElSherbiny et al. (1982) that is used in both Equations 12 and 23 given above is flawed, especially the trends exhibited between the data at aspect ratios of 40 and 80. The reason for this belief is presented below.

Before comparing the results of other studies with the results of ElSherbiny et al. (1982), a brief discussion of the general and individual empirical correlations presented in their paper is warranted. In their work, a general correlation equation (Equation 2 in this paper) covering the range of aspect ratios and Rayleigh numbers studied was developed and is currently used in the computer programs THERM and WINDOW4.1., ElSherbiny et al. (1982) also developed six correlation equations, each valid for an individual aspect ratio of 5, 10, 20, 40, 80, and 110. The authors believe that the six individual aspect ratio correlations represent the experimental data of ElSherbiny et al. (1982) more accurately than the general correlation given in Equation 2. Therefore, all of the subsequent discussions and comparisons in this paper will be made to ElSherbiny's individual aspect ratio correlation equations.

The numerical study by Korpela et al. (1982) showed that for 6000 < Ra < 17750, which is the typical fenestration design range, the Nusselt number at $A = 20$ is more than 10% higher than at $A = 40$. Similar observations were found in the experimental investigation of Yin et al. (1978). Their results showed that the cavity heat transfer rate (i. e., average Nusselt number) is 9.5% higher at $A = 20$ than t at $A = 40$ and 9.1% higher at $A = 40$ than at $A = 78$ for the Rayleigh number range between 1500 and 7,000,000 (7×10^6). The ElSherbiny et al. (1982) individual aspect ratio correlations give a maximum Nusselt number 13% higher at $A = 20$ than t at $A = 40$ for Ra = 7000

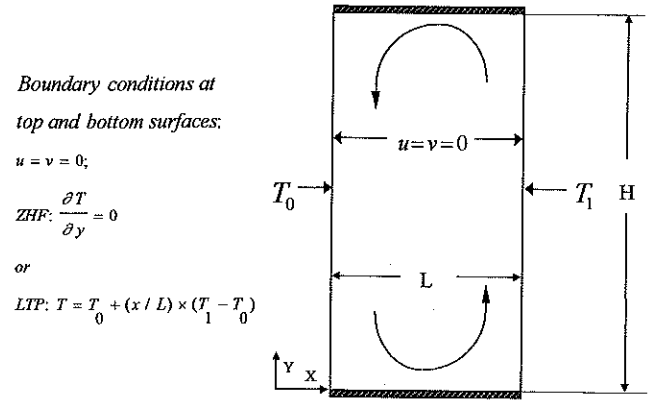

Figure 1 *Schematic geometry and boundary conditions for analysis of glazing cavity.*

and show a maximum Nusselt number 5.6% higher at $A = 80$ than at $A = 110$. These trends are in general agreement with the trends found in Korpela et al. (1982) and Yin et al. (1978). In other words, as the aspect ratio increases, the Nusselt number (or heat transfer rate) decreases.

However, the ElSherbiny individual aspect ratio correlations predict that at $A = 40$, the maximum Nusselt number is 5% lower than at $A = 80$, which is not in agreement with the results given in Korpela et al. and Yin et al. This anomaly also does not make physical sense, since at any fixed Rayleigh number, as the aspect ratio increases, the Nusselt number should decrease. Based on the above observation, by taking this to the limit of an infinite aspect ratio (a very tall cavity where the top and bottom flow-turning effects are negligible), for any fixed Rayleigh number, the Nusselt number should decrease toward the conduction-only limit (Nu = 1). It will be shown that the new correlation presented in this paper exhibits this trend. It also predicts that the Nusselt number always decreases at constant Rayleigh number as the aspect ratio increases, which agrees with the trends found in Korpela et al. (1982) and Yin et al. (1978).

Significant progress has been made during the past several decades in developing more accurate computing tools and methods, and, as a result of improved U-factor ratings, fenestration products are now more energy efficient than in the past. However, from the point of view of fundamental research, as well as improvement in the calculation methods used in THERM and WINDOW, it is desirable to resolve the dependence of the Nusselt number on the aspect ratio for the typical fenestration cavity design range. The object of this work is to contribute toward that end.

PROBLEM DEFINITION

The physical problem domain considered here is two-dimensional vertical air-filled (Pr = 0.71) rectangular cavities as shown in Figure 1. This type of configuration is typical of glazing cavities found in fenestration glazing units as well as in solar collectors.

The two vertical walls are held at constant temperatures of T_1 and T_0 with $T_1 > T_0$. The top and bottom walls are adiabatic, which is called zero heat flux (ZHF) boundary conditions. The fluid flow and heat transfer inside the cavity are governed by the continuity, momentum,and energy equations with the underlying assumption of Boussinesq approximation and an incompressible fluid flow with negligible viscous dissipation. Since no exact closed-form solution exists for this physical problem, a general-purpose fluid flow and heat transfer finite-element analysis software package FIDAP (FDI 1993) was used to obtain numerical solutions. Unlike the perturbation method, which introduces a set of linearized perturbation equations into the original governing equations and neglects the nonlinear terms in the resulting equations, the finite element method (FEM) used here directly solves the fully coupled system of nonlinear governing equations without the need for additional approximations or initial perturbation, and it has proved to be adequate to simulate the multicellular flow (Zhao et al. 1997b).

To obtain accurate results, the mesh employed in this study was developed using a repeated refinement technique, i.e., the thickness of the first layer of elements closest to the vertical wall must be checked with the formula $\delta = L(\mathrm{Ra}\frac{L}{H})^{-0.25}$ for estimating a boundary layer thickness. The mesh refinement should be undertaken whenever the thickness of the first layer of elements is larger than that of δ. Grid studies were performed extensively and have been reported by the authors in previous publications (Curcija 1992; Zhao et al. 1997a). For example, at an aspect ratio of 40 and Ra of 14200, a total of 9 mesh densities from 12 elements (across cavity thickness) by 80 elements (across cavity height) to 20 elements by 160 elements has been used to assess the simulation accuracy. It was found that 16 elements by 128 elements is sufficient to obtain an accurate value—the Nu value obtained was 1.402, which agrees well with 1.417 of ElSherbiny's (1982) experimentally derived correlation and 1.38 of Lee and Korpela's (1983) numerical calculation at $A = 40$ and Ra = 14200. In addition, the convergence criteria of 1×10^{-4} for both solution vector and residual vector were used in order to ensure the simulation accuracy. For those who are interested in the formulation of the governing equations as well as the numerical solution procedures, detailed depictions can be found in Curcija (1992) and Zhao et al. (1997a).

NEW HEAT TRANSFER CORRELATION

A total of 15 aspect ratios, from 5 to 110, and Rayleigh numbers within the typical fenestration glazing cavity design range (Ra $\leq$ 20000) were investigated. Sample calculated results are shown in Figure 2. It can be observed that at each aspect ratio, extensive numerical calculations were carried out

that are well distributed over the entire fenestration design range and are essential for developing an accurate heat transfer correlation. In addition, the results clearly show a significant aspect ratio effect on the averaged Nusselt number. Using the nonlinear fit algorithm given in Wolfram (1992), the results obtained in this study were correlated into the following equations for $A = 5$ to $A = 110$:

$$Nu_1 = \left[1 + \left(\frac{0.788335 \cdot \left(\left(1.42227 - \frac{1.41845}{A}\right)\frac{Ra}{A}\right)^{0.881073}}{139.677 + \left(\left(1.42227 - \frac{1.41845}{A}\right)\frac{Ra}{A}\right)^{0.724505}}\right)^2\right]^{0.5} \quad for 5 \le A < 30 \tag{4a}$$

$$Nu_2 = \left(1 + 0.00044265 \cdot \left(\frac{Ra}{A}\right)^{1.36869}\right)^{0.326071} \quad for 30 \le A \le 110 \tag{4b}$$

It can be observed that Equations 4a and 4b are functions of both aspect ratios and Rayleigh numbers for the entire investigation range, e.g., $5 \le A \le 110$ and $Ra \le 20000$. In addition, the forms of both correlation equations are more comprehensive than that of single power law type equations; therefore the new heat transfer correlations can account for the complex interaction of aspect ratios and Rayleigh numbers on averaged Nusselt numbers. Comparisons were made between this new correlation and the individual aspect ratio correlations of ElSherbiny et al. (1982) as shown in Figures 3 to 7. In Figure 3, for an aspect ratio of 5, the new correlation, Equation 4a, gives higher Nusselt numbers than ElSherbiny's correlation. The reason for this is the effect of different thermal boundary conditions, the zero heat flux (ZHF) condition of the current study versus the linear temperature profile (LTP) condition of ElSherbiny et al. (1982)) on the top and bottom cavity surfaces. As indicated by both Raithby and Wong (1981) and Zhao et al. (1997a), the value of the heat transfer rate is lower for the LTP than the ZHF case for low aspect ratios, and this difference decreases as the aspect ratio increases.

This trend is clearly illustrated in by the comparisons presented in Figures 3 to 5. In Figures 4 and 5, for aspect ratios of 10 and 20, very good agreement was found between the new correlation (Equation 4a) and ElSherbiny's individual correlation equations. For an aspect ratio of 40, the discrepancies that were previously discussed are observed in the comparisons shown in Figures 6 to 8. In addition, ElSherbiny's correlations at $A = 40$, 80, and 110 all depart from the conduction regime (where the slope changes significantly) around the same value of the Rayleigh number ($Ra = 5000$) and increase exponentially from that point.

This behavior of ElSherbiny's correlation is in conflict with some fundamental experimental and analytical studies. Batchelor (1954) first analytically defined the conduction regime, where heat is transferred across the cavity primarily

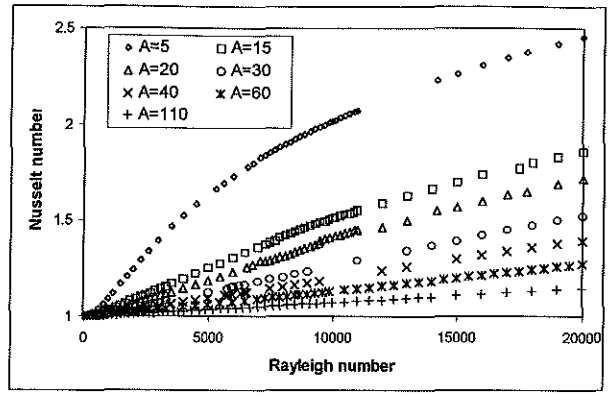

Figure 2 *Sample calculation results for different aspect ratios.*

by conduction, and the boundary layer regime, where the flow is confined to boundary layers at the side walls for which the dominant mode of heat transfer is convection. It should be noted that in the conduction regime, the dominant mode of heat transfer is conduction; however, convection does contribute slightly to the energy transfer at both ends of the cavity where flow turns. As a result of this combined effect, the Nusselt number is greater than the pure conduction heat transfer rate of $Nu = 1.0$. As the Rayleigh number increases (i.e., thermal loading increases), the Nusselt number also increases, but not significantly as long as flow stays in the conduction regime. Batchelor also concluded that the higher the aspect ratio, the wider the range of the conduction regime. Based on Batchelor's analytical study, the limit of transition from the conduction regime to the boundary layer regime at $A = 40$ is $Ra = 20000$ and extends to $Ra = 55000$ at an aspect ratio of 110.

The existence of the conduction regime and the boundary layer regime were later confirmed by the interferometer study of Eckert and Carlson (1961), and the trend that the range of the conduction regime increases as the aspect ratio increases was experimentally observed. Eckert and Carlson also further refined the regime definitions as the *conduction regime*, the *transition regime*, and the *boundary layer regime*. As a result of these newly defined regimes, the limit for the flow leaving the conduction regime in the study of Eckert and Carlson (1961) is smaller than that of the analytical study of Batchelor (1954). However, the Rayleigh numbers at the limit of transition for $A \ge 40$ are still much higher than ElSherbiny's departure points (where the slope changes significantly) shown in Figures 6 to 8 at $Ra = 5000$. Furthermore, for $A \ge 42$, turbulence does not exist until $Ra \ge 13700$ (Batchelor 1954) and multicellular flow (where convective effects start to become more significant) begins to form after a Ra of 5950 (Zhao et al. 1997b). These multicellular flow effects will increase heat transfer rate (or Nusselt number) slightly (Wright 1990) and there is no obvious reason why ElSherbiny's correlations predict significantly higher values. In addition, as discussed previously, contrary to the trend that at fixed Rayleigh number

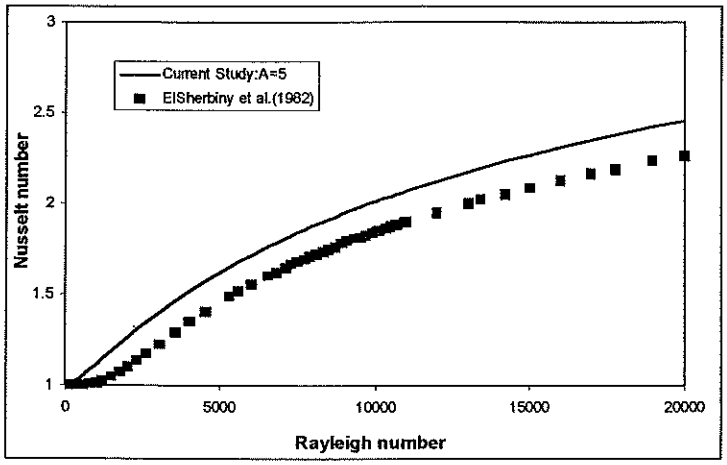

Figure 3 *Comparison of current calculation with experimental results of ElSherbiny et al. (1982) for A=5.*

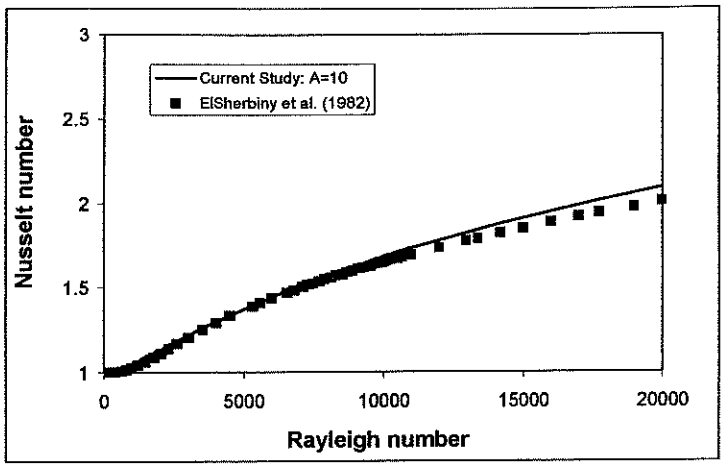

Figure 4 *Comparison of current calculation with experimental results of ElSherbiny et al. (1982) for A=10.*

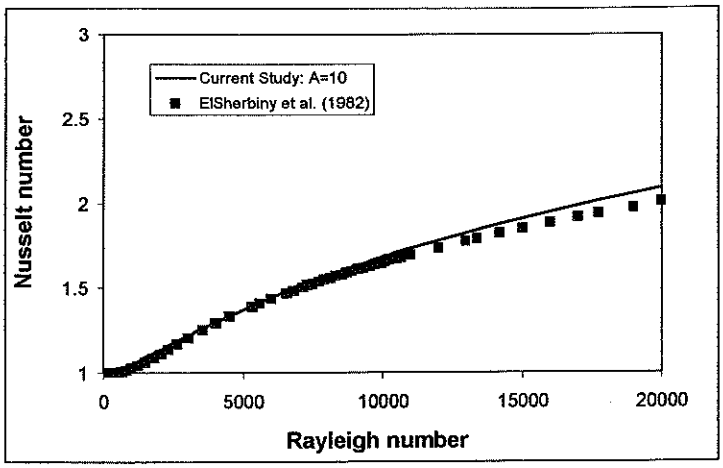

Figure 5 *Comparison of current calculation with experimental results of ESherbiny et al. (1982) for A=20.*

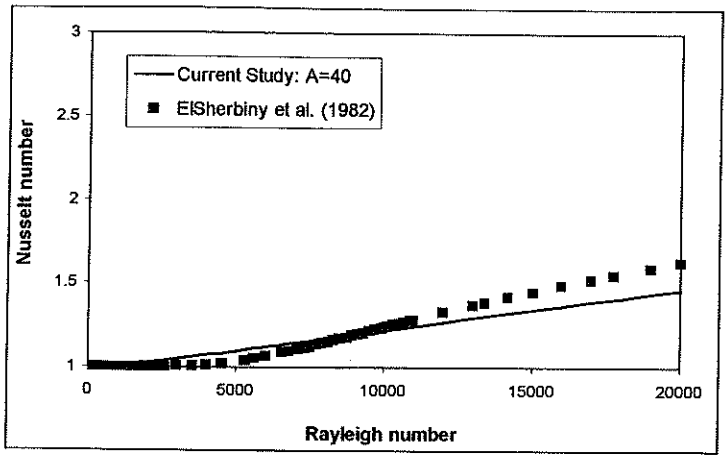

Figure 6 *Comparison of current calculation with experimental results of ElSherbiny et al. (1982) for A=40.*

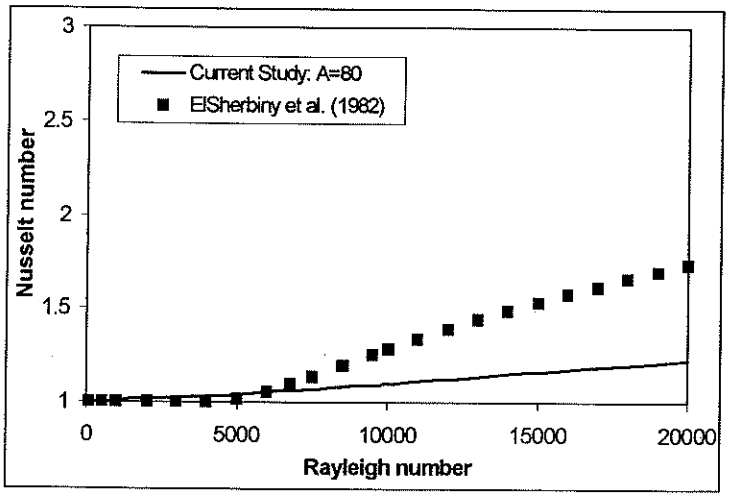

Figure 7 *Comparison of current calculation with experimental results of ElSherbiny et al. (1982) for A=80.*

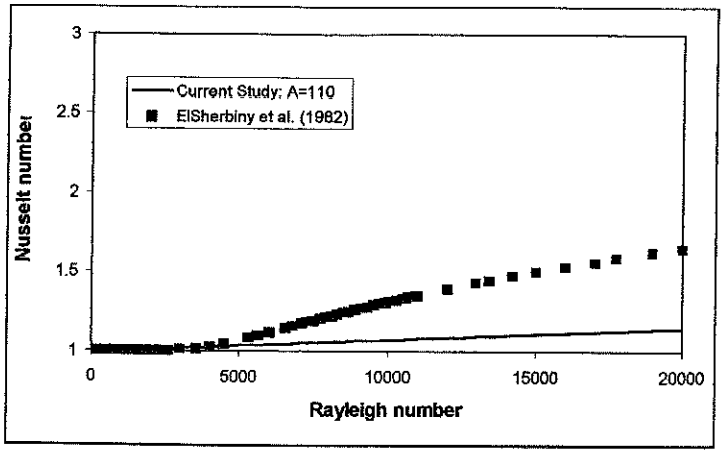

Figure 8 *Comparison of current calculation with experimental results of ElSherbiny et al. (1982) for A=110.*

the Nusselt number should decrease as the aspect ratio increases, ElSherbiny's correlation for $A = 80$ predicts larger Nusselt numbers than the correlation for $A = 40$ does between Rayleigh numbers from 7000 to 20000. Since this portion of ElSherbiny's experimental data is questionable, it is important to recognize that the current glazing cavity heat transfer correlation (Equation 3) adopted by ASHRAE SPC-142P is partially based on the experimental data of ElSherbiny et al. (1982).

CONCLUSIONS AND RECOMMENDATIONS

In this study, a new set of convective heat transfer correlations (Equations 4a and 4b) were developed, which resolves the dependence of the averaged Nusselt number on both the aspect ratio and the Rayleigh number over the typical fenestration cavity design range. The new correlation compares favorably with ElSherbiny's "fine-grain structure" for $A < 40$, and it is believed to be accurate over the whole investigative range of this study because its trends agree well with the classical flow regime definitions. For aspect ratios greater than 40, it was found that problems apparently exist in the experimental data presented in ElSherbiny et al. (1982). Future experimental and analytical research should be conducted to fully resolve this issue and to extend the new correlation to include turbulent flow results at higher Rayleigh numbers.

NOMENCLATURE

A	= aspect ratio, H/L
H	= height of air layer (m)
L	= thickness of air layer (m)
T_1	= constant temperature at cavity hot wall (K)
T_0	= constant temperature at cavity cold wall (K)
Nu	= Nusselt number, hL/k
Pr	= Prandtl number = 0.71 for air, ν/α
Ra	= Rayleigh number, $g\beta\Delta TL^3/\nu\alpha$
ZHF	= zero heat flux boundary conditions
LTP	= linear temperature profile boundary conditions
d	= boundary layer thickness

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